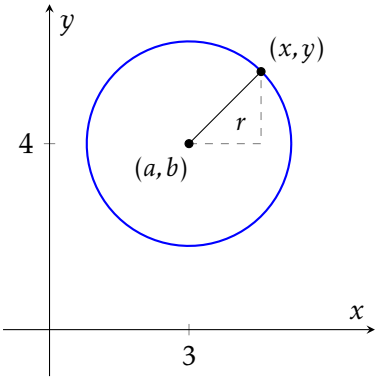
# The Equation of a Circle

A circle is the set of points at a fixed distance $r$ from a fixed centre. The equation comes straight from Pythagoras.

**Theorem**

Centre $(0,0)$, radius $r$ : $\quad x^2 + y^2 = r^2$

Centre $(a,b)$, radius $r$ : $\quad (x-a)^2 + (y-b)^2 = r^2$




**Example** 1. Write down the centre and radius of $(x-3)^2 + (y+5)^2 = 49$.

2. Find the equation of the circle with centre $(2,-1)$ passing through $(4,-5)$.

3. Does the point $(7,1)$ lie inside, on, or outside the circle in part 2?

**Example**

The points $A(-2, 11)$ and $B(8, 1)$ are the ends of a diameter of a circle. Find the equation of the circle.

**Textbook Exercises:** SPS Course 3.4, Exercise 0

## The General Form

Expanding $(x-a)^2+(y-b)^2=r^2$ gives an equation of the form

$$x^2+y^2+2gx+2fy+c=0.$$

To find the centre and radius, complete the square in $x$ and in $y$.

**Example** (Edexcel C2)
The circle $C$ has equation $x^2+y^2-20x-24y+195=0$. Find the centre and radius of $C$.

**Example**
Show that the circle $(x-3)^2+(y-5)^2=4$ touches the circle $x^2+y^2-2y-8=0$.

**Example**

Find the values of $k$ for which $x^2 + y^2 + 6x - 4y + k = 0$ is the equation of a circle.

**Textbook Exercises:** SPS Course 3.4, Exercise 1

## Tangents and Normals

**Fact** — The tangent to a circle at a point $P$ is perpendicular to the radius at $P$. So:

$$\text{gradient of tangent at } P = -\frac{1}{\text{gradient of radius to } P}$$

The normal at $P$ is the line through the centre.

**Example** (Edexcel C2)

A circle $C$ with centre $(2,-1)$ passes through the point $A(4,-5)$.

1. Find an equation for $C$.
2. Find an equation of the tangent to $C$ at $A$, in the form $ax+by+c=0$ for integers $a$, $b$, $c$.

**Fact** (Length of a tangent from an external point) — If the tangent from an external point $N$ touches the circle (centre $M$, radius $r$) at $P$, then triangle $MPN$ is right-angled at $P$:

$$NP^2 = MN^2 - r^2$$

**Example** (Edexcel C2)

The circle $C$ has equation $x^2 + y^2 - 20x - 24y + 195 = 0$, with centre $M$. The tangent to $C$ at a point $P$ passes through $N(25, 32)$. Find the length $NP$.

**Textbook Exercises:** SPS Course 3.4, Exercise 2

## Lines Meeting Circles

To find where a line meets a circle, substitute the line into the circle. The discriminant of the resulting quadratic counts the intersections:

$b^2 - 4ac > 0$ the line crosses the circle twice
$b^2 - 4ac = 0$ the line is a tangent
$b^2 - 4ac < 0$ the line misses the circle

**Example**

Find the points where the line $y = x + 1$ meets the circle $x^2 + y^2 = 25$.

**Example**

Find the values of $k$ for which the line $y = 7x + k$ never intersects the circle $x^2 + y^2 = 2$.

**Exercise** (Investigation)**.** Find the equation of the (unique) circle passing through $(0, 0)$, $(6, 0)$ and $(0, 8)$. Then devise a general method for finding the circle through any three given points, and identify when no such circle exists.

**Textbook Exercises:** SPS Course 3.4, Problems 3.4.3